ATMOSPHERIC WATER VAPOR PROFILES DERIVED FROM REMOTE-SENSING RADIOMETER MEASUREMENTS

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ABSTRACT

The feasibility and preliminary testing of a low cost, remote-sensing air-borne, double bolometer technique for inferring atmospheric water vapor is illustrated. To deduce the water vapor profile with commercially available equipment, the radiative transfer equation is solved for the water vapor transmissivity employing an input data remote radiometer-measured upward irradiances obtained at aircraft holding levels. Radiometers sensitive in two separate spectral bands are used. The primary radiometer covers the 4.39 to 20.83μ, broad atmospheric radiation band, and the second, for surface temperature deduction, covers the atmospheric window region, 7.35 to 13.16μ.

The transfer solution results are acquired from computer programs developed specifically for this purpose. Results indicate an accuracy for inferred total tropospheric water vapor and mixing ratio profiles close to that of the standard sounding electrical hygrometer. The absolute accuracy of the radiosonde hygrometer, considering surface calibration procedures, and for a single ascent, is not better than ± 12 percent. The absolute accuracy is greatest for "dry" soundings where the largest changes in irradiance occur for given changes in moisture.

Specifically, tests for a vertical profile averaging 6.00 gm./kg. of water vapor produce an average error of 0.70 gm./kg. in the inferred mixing ratio. The average error in mixing ratio obtained by this technique for profiles averaging 2.3 gm./kg. is 0.05 gm./kg. The implications for use on high-flying aircraft or on rockets with highly sensitive radiometers are obvious. The primary purpose in reporting this research is to suggest a technique and illustrate its use. It is clear that with more sensitive bolometer radiometers with selective band pass filters a considerable increase in accuracy can be achieved.

LIST OF SYMBOLS

$F\uparrow$	Upward irradiance (watts/meter^2)
i, j	Subscripts employed in indexing variables
$\Delta\nu$	Wave number increment (centimeter^{-1})
B	Black body irradiance (watts/meter^2)
ν	Wave number (centimeter^{-1})
T	Temperature (°C.)
p	Pressure (millibars)
Φ	Filter transmissivity (percent)
$\Delta\tau$	Transmissivity increment (percent)
w	Water vapor quantity
C	Carbon dioxide quantity
0	Subscript denotes surface value of subscripted parameter
W	Mixing ratio (grams/kilogram)
λ	Exponent of a power law equation
u^*	Optical depth of atmospheric gas (grams/centimeter^2), pressure and temperature scaled
$\hat{F}\uparrow$	Calculated upward irradiance (watts/meter^2)
F	Subscript denoting irradiance
L	Generalized absorption coefficient for water vapor after Elsasser [9]
m	Limiting wave number increment identifier
n	Number of atmospheric levels between surface and a reference level
T_{eq}	Black body equivalent radiating temperature
τ	Transmissivity (percent)

1. INTRODUCTION

The radiative power transfer equation can be solved iteratively to infer atmospheric water vapor from remote radiant power measurements. The purpose of this research is to propose and describe this technique for deducing water vapor quantities employing observations of radiant power as input to the radiative power transfer equation. With more sensitive bolometer radiometers having selective band pass filters greater accuracy than is reported can be obtained. On-hand equipment was used in this pilot study.

The observations were made in two spectral regions, 4.39 through 20.83u and 7.35 through 13.16μ. Results of a similar technique employing balloons were given by Kuhn [1] and Kuhn and Cox [2]. The technique differs from those of satellite radiometric inferences of atmospheric water vapor and temperature (Houghton [3]; Wark [4]; Möller [5]; King [6], [7]). The latter observations are necessarily limited to observations from a fixed satellite orbit. They employ highly sensitive sensors receptive to radiant power in one or more different spectral intervals.

This procedure requires measurements of temperature, height, and spectral irradiance at a number of aircraft holding levels. Two "shelf-type" radiometers sensitive over two different spectral intervals constitute the sensor capability. One radiometer monitors the air-surface interface temperature in the relatively transparent position of the atmospheric spectrum, while the other measures the spectral component of upward terrestrial long-wave irradiance as a function of height in the broad-band, earth-atmosphere, self-emission region. Ideally, this second radiometer would be sensitive over the strongly absorbing atmospheric water vapor band centered at 6.7μ. Funding and ready availability prevented use of such a special purpose bolometer radiometer.

2. RADIANT POWER COMPUTATIONS

As mentioned, pressure, temperature, and water vapor data are the normal input to the radiative transfer equation solution for spectral irradiance passing upward through the atmosphere.

For a plane parallel atmosphere, containing no scatterers, in local thermodynamic equilibrium and consisting of gaseous water vapor and carbon dioxide, the upward irradiance through any reference level above the surface may be obtained by the following finite difference form of the radiative transfer equation,

$$F\uparrow=-\sum_{j=1}^{m}\Delta\nu_j\sum_{i=1}^{n}B_i(\nu,\overline{T}(p))\Phi(\nu)\Delta\tau_i(\nu,u^*(p,w),T)$$

$$-\sum_{j=1}^{m}\Delta\nu_j\sum_{i=1}^{n}B_i(\nu,\overline{T}(p))\Phi(\nu)\Delta\tau_i(\nu,u^*(p,C),T)$$

$$+\sum_{i=1}^{m}B_i(\nu,T(p_0))(\tau_i(\nu,u^*(p,w),T)\tau_i(\nu,u^*(p,C),T.)\quad(1)$$

The effects of other radiatively active gases such as ozone and nitrous oxide in the wavelength regions considered are insignificant, amounting to less than 1 percent of the upward irradiance, and are not considered.

Equation (1) is employed in deducing the quantity of atmospheric water vapor from measurements of upward irradiance for a particular spectral interval. An iteration procedure resulting in a direct solution of the water vapor transmissivity, $\tau_w(\nu, p, T)$, is used. Since the water vapor transmissivity is a function of the quantity of water vapor, w, beneath a given reference level, and since the amount of water vapor is a function of the mixing ratio, the solution yields the mixing ratio.

The iteration procedure requires calculations of irradiance with a stepped series of trial values of water vapor (equation (1)) until the difference between calculated and observed upward irradiance is minimized. Carbon dioxide emission and transmission are calculated assuming a constant mixing ratio. A first approximation for the water vapor profile is described by a power law expression of the form,

$$W=W_0(p/p_0)^\lambda\quad(2)$$

after Smith [8]. λ, the exponent of a power law expression, changes the water vapor profile as required in the iteration. For tropical soundings W_0 is assumed to be 5.0 gm./kg., certainly a lower bound. For mid-latitude summer soundings, W_0 is assumed to 0.6 gm./kg. For mid-latitude winter soundings W_0 is assumed to be 0.06 gm./kg. In other words, a lower bound is chosen to start the computations.

The iteration procedure, involving repeated solutions of equation (1), requires minimizing the quantity,

$$\sum_i(F_i\uparrow-\hat{F}_i\uparrow)^2.$$

Stepwise changes in λ, (equation (2)) for subsequent successive increases in W_0 provide the repeated input of the "trial" water vapor quantities required for equation (1). The computer evaluates $F_i\uparrow$ from an initial "minimum" profile of W_0 shaped by an initial λ value of 0.5. The λ values are stepped upward to a maximum value of 3.0 and then the process repeats with a new stepped increase in W_0. Negative values of λ will allow the mixing ratio profile to increase with altitude above the surface. The changing of the entire water vapor profile by the power function approximation of equation (2) imposes a stabilizing constraint on the solution of equation (1). This same procedure has been used by Kuhn and Cox [2] to infer stratospheric water vapor profiles. It should be noted that the constraint of equation (2) does not preclude convergence at every level, within reason, since the stepwise variation of λ allows, literally, almost any characteristic shape to the W profile to obtain. Average computer solution time is 0.3 min. (CDC–1604) for 10 levels, over the spectral range 4.39 to 20.83μ (480–2280 cm.$^{-1}$).

3. UNIQUENESS OF THE SOLUTION FOR INFERRED WATER VAPOR

In view of the rate of change of the water vapor slab transmissivity, τ_F, with changes in the amount of atmospheric water vapor, u^*, it is necessary to establish the uniqueness of the radiometrically inferred water vapor quantity. Figure 1 gives the water vapor slab or irradiance transmissivity as a function of the sum of the logarithm of u^*, the pressure and temperature scaled optical thickness, and the logarithm of the generalized absorption coefficient L, (Elsasser [9]). This is expressed by,

$$\tau_F=\tau_F\,(\log_{10}u^*+\log_{10}L).\quad(3)$$

From this figure it is evident that the greatest change in transmissivity for a given change in $\log u^*+\log L$ occurs between values of -1.50 and 0.50 for $\log u^*+\log L$. Assuming a mean value for $\log L$ of -0.50, this represents

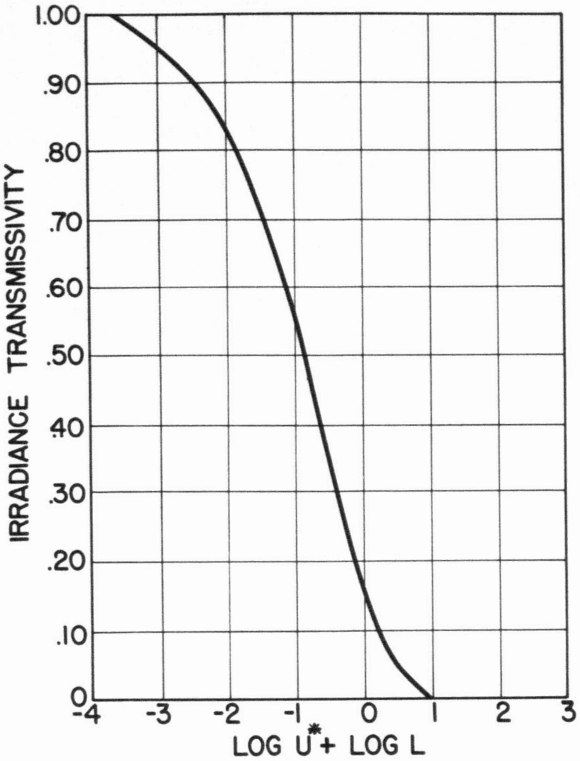

FIGURE 1.—Water vapor beam transmissivity vs. $(\log_{10}u^*+\log_{10}L)$.

an optical thickness range of from approximately 0.1 gm./cm.² to 10.0 gm./cm.²

Aerosol contamination is clearly evident as a sharp discontinuity in the observed moisture profile. In fact, detection of thin clouds at night is possible with these instruments.

4. IRRADIANCE OBSERVATIONS

The primary instrument employed in this research was a chopper bolometer radiometer having a 30° optical field of view, germanium optical flat-front lens with a spectral bandpass from 4.39 to 20.83μ (480 to 2280 cm.$^{-1}$). The mean band transmissivity is 0.50.

The relative resolution and special characteristics of the primary radiometer used in these experiments are given in table 1. Various optical companies can provide bolometer radiometers with resolution of at least 0.025°C., twice the resolution of the unit used. Thus for a 0.025°C. change in target temperature at 5.0°C. the corresponding

TABLE 1.—*Radiometer specifications*

Spectral passband	4.39 to 20.83 μ.
Average filter transmissivity	0.58
Field of view	30°.
Maximum temperature resolution	0.05°C.
Irradiance change behind germanium flat filter for 0.05°C. change at 5.0°C.	0.7 micro watt/cm.² (.007 watt/m.²)
Temperature range	−40.0°C. to +30.0°C.
Recorder output	0–50 millivolts full scale into 1000 ohms

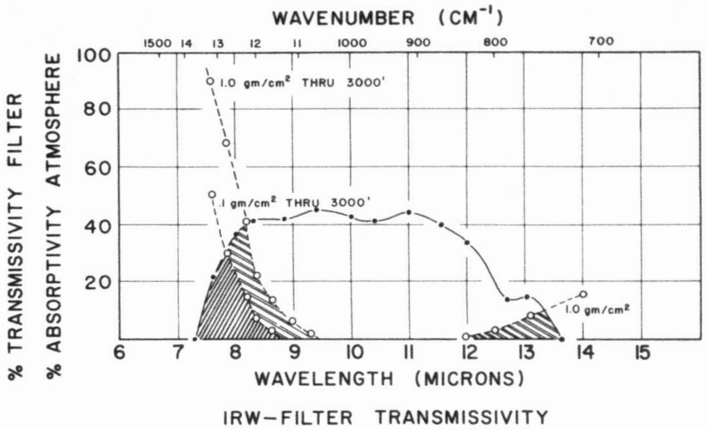

FIGURE 2.—Window radiometer filter transmissivity vs. wavelength.

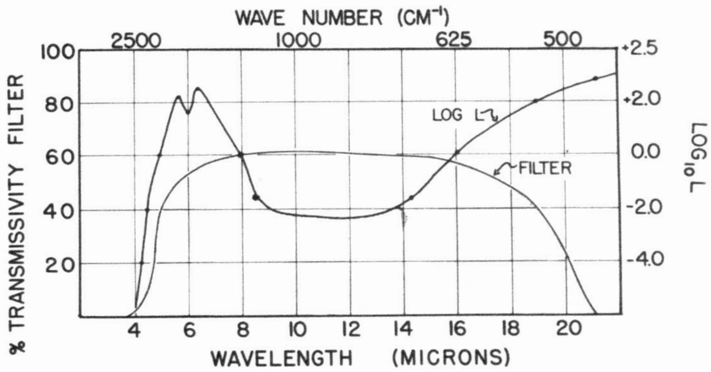

FIGURE 3.—Broad-band radiometer filter transmissivity vs. wavelength. $\log_{10}L$ vs. wavelength.

irradiance change behind the optical flat filter is 0.0035 watt/m.² For an average tropospheric sounding to 20,000 ft., this would correspond to a mean water vapor mixing ratio resolution of 0.05 gm./kg. or 50 parts per million for a moderately dry atmosphere. Throughout, we are considering air-borne radiometers at costs not exceeding $10,000. The absolute accuracy of the broad bandpass radiometer is 0.028 watt/m.² The absolute accuracy of the "window" radiometer is 2°C. above 0°C.

The curves of the filter transmissivity of the "window" radiometer and the primary broad-band earth and atmosphere self emission radiometer are shown in figures 2 and 3. The solid angle aperture of the surface temperature monitor "window" radiometer is 3°. The radiometers were calibrated against a hemispherically symmetrical black source. We then have,

$$F]_{\nu_1}^{\nu_2}=\int_{\nu_1}^{\nu_2} B(\nu,T_{eq})\Phi(\nu)d\nu. \tag{4}$$

The equivalent blackbody temperature T_{eq}, can be determined from equation (4). $B(v,T_{eq})$ is the hemisphere blackbody irradiance. The curves of radiometer power versus equivalent blackbody source temperature are

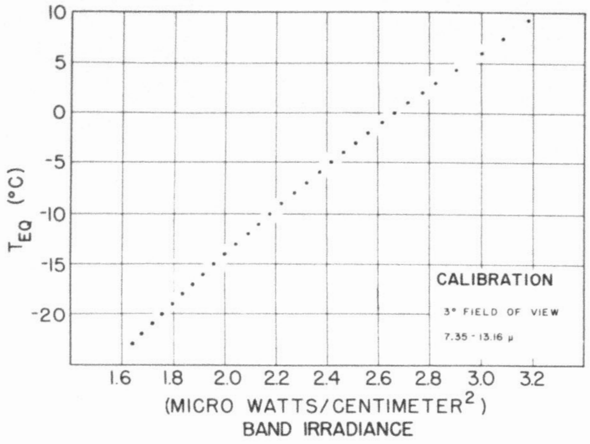

FIGURE 4.—Window radiometer calibration curve, equivalent blackbody temperature vs. micro watts/square centimeter.

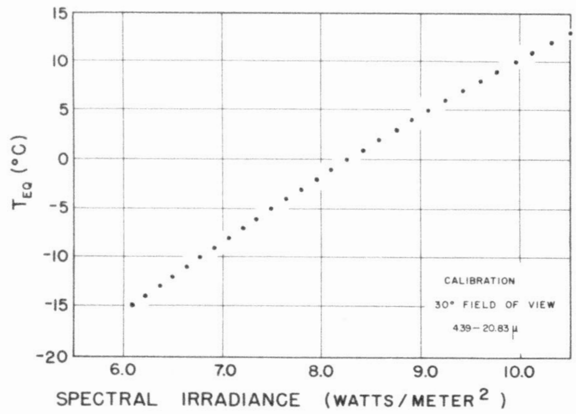

FIGURE 5.—Broad-band radiometer calibration curve, equivalent blackbody temperature vs. watts/square meter.

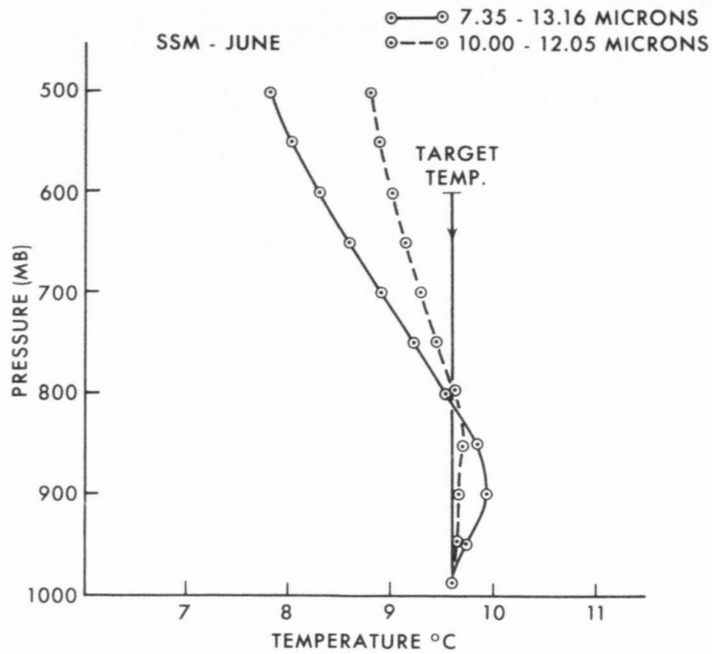

FIGURE 6.—Temperature correction vs. pressure at Sault Ste. Marie, Mich. (June) for radiometers equipped with 7.35–13.16-μ filter and a 10.00–12.05-μ filter.

reproduced in figures 4 and 5 for the window and broad bandpass radiometers, respectively.

Reference to figure 2 shows the considerable overlap of the water vapor absorption band in the bandpass area, 7.1 to 9.5μ. To show the closing effects of this band, irradiance calculations for received power were run for the June mean monthly sounding at Sault Ste. Marie, Mich., from sea level to 18,000 ft. Figure 6 illustrates the influence of water vapor absorption for this sounding in the shorter-wave end of the filter used on the window radiometer (7.35–13.16μ), and the lesser influence in the 10.00 to 12.05-μ bandpass of a currently available "window" radiometer. The deduced surface temperature error for the narrow pass filter is about half that of the window radiometer used in this experiment when observing the surface at 700 mb. (or 10,000 ft.). In addition, a temperature and moisture inversion below 900 mb. has considerably less effect in the narrower bandpass region. These facts are, of course, not new, but in light of the fact that IR radiometers are in such heavy use today in a variety of disciplines, this point deserves this amplification.

5. AIRCRAFT SOUNDINGS

The NCAR Queenaire Beechcraft was made available for this research by Dr. D. R. Rex, Director of Flight Facilities at National Center for Atmospheric Research. Viewing ports with shock mountings supported the two radiometers. Simultaneous measurements of the two upward irradiances, altitude, and air temperature were made coincidentally with visual observations of the surface.

One aircraft sounding was made in visually determined cloudless conditions on July 14, 1965. The area surveyed was over Lake Superior, 20 mi. east of Duluth, Minn., covering the period 2245 CDT through 2331 CDT. Table 2 gives the observed pressure, air temperature, surface temperature, mixing ratio, and upward flux. In addition it displays the calculations of upward irradiance and the inferred mixing ratio vertical profile. The iteration on mixing ratio converges quite rapidly.

Table 3 gives the results of a second ascent made in a light aircraft, to approximately 9,000 ft., on May 25, 1966, just east of Green Bay, Wis., but not over Lake Michigan. The atmosphere was fairly moist, with an optical thickness of 2.1 gm./cm.^{2} through 10,000 ft.

The several tabulations presented enable a comparison of measured and calculated irradiances and the inferred water vapor profile in gm./kg. The errors, shown in the last column, are small enough to make these remotely sensed inferred water vapor measurements useful.

TABLE 2.—*Observed and calculated sounding data, Duluth, Minn., July 14, 1965*

Height (ft.)	Air Temp. (°C.)	Observed Mixing Ratio (gm./kg.)	Observed Irradiance (w./m.^{2})	Mean Calculated Irradiance (w./m.^{2})	Inferred Mixing Ratio (gm./kg.)
0	13.5	9.2	10.410	10.410	8.3
300	19.4	9.2	10.490	10.490	8.3
2,600	16.3	8.1	10.714	10.719	7.3
3,600	13.3	7.5	10.032	10.047	6.8
5,600	7.9	6.4	9.556	9.567	5.8
6,600	5.4	6.0	9.259	9.270	5.4
7,600	3.8	5.2	9.079	9.091	4.7
8,600	1.8	4.7	8.929	8,940	4.4
10,600	0.3	3.5	8.747	8.771	4.1

Columnar optical thickness is 2.035 gm./cm.^{2}

TABLE 3.—*Observed and calculated sounding data, Green Bay, Wis., May 25, 1966*

Height (ft.)	Air Temp. (°C.)	Observed Mixing Ratio (gm./kg.)	Observed Irradiance (w./m.^{2})	Mean Calculated Irradiance (w./m.^{2})	Inferred Mixing Ratio (gm./kg.)
0	10.0	6.5	9.760	9.760	3.00
1,385	16.0	2.8	9.716	9.721	2.80
[illegible],250	9.0	2.2	9.761	9.761	2.20
4,778	7.0	2.3	9.465	9.467	2.20
7,740	−1.0	2.00	9.110	9.115	2.00

Columnar optical thickness is 0.55 gm./cm.^{2}

6. CONCLUSIONS

Some relatively expensive infrared hygrometers and refractometers are available for aircraft observations of water vapor. However, their power requirements and operations techniques are much more severe than those for the radiometric technique that we have discussed. Applications to remote sensing of the atmospheric moisture structure in remote areas without recourse to on-surface observations appears feasible in the manner discussed.

Since the greatest resolution occurs in that part of the water vapor spectrum between 0.1 and 10.0 gm./cm.^{2} of water vapor, it is possible that a rocket-borne, upward-facing radiometer could provide high-altitude moisture data by observing irradiance deviations from a constant, cold background. The radiometer could measure downward irradiance during descent from altitudes to 80 km. With sufficiently high relative radiometric resolution, one could monitor the downward component of spectral irradiance against a steady background of "zero" radiant power. Certainly this is not a new idea, but appears worth investigation. The results could augment high-altitude frost-point equipment such as that of Mastenbrook [10]. Chopper bolometer radiometers could be replaced with selectively sensitized thermistor detectors which can survive the accelerations of small rocket launch.

A further application of the infrared for remote sensing inference of total atmospheric water vapor content from the surface to a prescribed level without the requirements of actual aircraft sounding was suggested by Suomi [11] after King [12]. This technique is being investigated by the authors. Briefly, this approach involves the sensing of the upward component of irradiance at a fixed level but at several different nadir angles. The primary measurements would be made in the water vapor vibration-rotation band centered 6.3μ (1587 cm.$^{-1}$) and in the window region, 10–12μ. The atmosphere beneath flight level is assumed horizontally homogeneous. Thus a choice of several nadir viewing angles would result in an equal number of "air masses," giving optical thickness of u^* and multiples of u^*. The irradiances sensed at the different angles are functions of an assumed lapse of temperature. The irradiances are also a function of the unknown optical masses u^* and multiples of u^*. Since one measures a difference in the irradiances directly, and since, although the absolute value is unknown one does know the relationship bet veen the magnitudes of the optical depths of the various atmospheric columns observed at the different nadir angles, a set of equations exists which enables a solution of the absolute magnitude of u^*, the total optical depth. This technique, an adjunct to this study, offers interesting possibilities when only the total mass of water vapor in the atmospheric column is desired such as in climatic surveys over inaccessible areas.

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